Generation of Plausible Hurricane Tracks for Preparedness Exercises

Jonathan K. Su, Charles Rose, Anthony Lapadula, Robert Hallowell
MIT Lincoln Laboratory, 244 Wood Street, Lexington, MA 02421
[su|c.rose|lapadula|bobh]@ll.mit.edu

Abstract—Under the National Hurricane Program, MIT Lincoln Laboratory is developing HURREVAC eXtended (HVX), which is replacing the legacy evacuation decision-support tool HURREVAC. The platform is designed to enable emergency managers (EMs) make accurate and timely evacuation decisions for their community by providing planning, situation awareness, data analytics, and training tools in a Web browser. HVX provides the novel capability to simulate storm tracks in a Web application with minimal user input or meteorological expertise. This capability will allow EMs to train more frequently and inexpensively, simulate storms that EMs have not already seen, and enable serious games and training analytics to improve EMs preparedness. The simulator leverages the HURDAT2 database, which contains information on more than 2,800 tropical storms, to generate plausible storms. The intended purpose and the required storm characteristics drove simulator design and technique selection. Storm movement between waypoints is simulated by a Markov model and kernel density estimation (KDE) with a gamma-von Mises product kernel. KDE with a beta kernel generates maximum sustained winds, and linear regression simulates minimum central pressure. Maximum significant wind extents are simulated by Poisson regression and temporal filtering. The un-optimized MATLAB code runs in less than a minute and is integrated into HVX as a Java package.

I. INTRODUCTION

Hurricanes and typhoons— tropical cyclones with maximum sustained winds of 64 knots (74 mph) or greater [1]—can cause many injuries and fatalities as well as millions or billions of dollars in damages [2]. Tropical cyclones are usually categorized by their maximum sustained winds according to Table I, with Category 3 and higher storms considered "major." Storm surge, inland flooding, and extreme winds cause the vast majority of fatalities and destruction [2].

Emergency managers (EMs) are responsible for making hurricane-related evacuation recommendations to higher officials. If ordered, an evacuation must be completed before the onset of tropical storm-force (34-knot) winds. An EM must coalesce information about the community's hurricane evacuation plans, National Hurricane Center (NHC) weather forecasts, and information from local National Weather Service (NWS) offices; coordinate with emergency services, neig-

TABLE I
TROPICAL CYCLONE STRENGTHS

Abbreviation	Strength	Maximum Sustained Winds [knots]
TD	Tropical Depression	[0, 33]
TS	Tropical Storm	[34, 49]
STS	Strong Tropical Storm	[50, 63]
HU1	Category 1 Hurricane	[64, 82]
HU2	Category 2 Hurricane	[83, 95]
HU3	Category 3 Hurricane	[96, 112]
HU4	Category 4 Hurricane	[113, 136]
HU5	Category 5 Hurricane	[137, ∞*)

*The Saffir-Simpson Hurricane Wind Scale (SSHWS) [3] does not specify an upper limit on the maximum sustained winds of a Category 5 hurricane, but the simulator in this paper imposes an upper limit of 180 knots (207 mph).

hboring governments, and relief agencies; and communicate with the public.

A. National Hurricane Program

Following Hurricane Katrina in 2006, the U.S. Government mandated the DHS S&T/FEMA National Hurricane Program (NHP). In 2014, NHP launched a Technology Modernization (TM) effort, which identified technology gaps, explored new solutions, established a Working Group (WG), and started new technology development [4].

For many parts of the continental U.S., the hurricane return period exceeds eight years, and the major-hurricane return period exceeds three decades [2]. The last storm to make landfall as a major hurricane in the continental U.S. was Wilma in 2005. Consequently, an EM might be familiar with the few historical storms that affected his or her community but have limited real-life evacuation experience.

Limited travel budgets, training sessions, and availability of NHC experts mean that less than 10% of EMs receive training each year. Sessions last 1–5 days, focus on hurricane forecasts and other NHC products, and cover only one or two scenarios. As a result, an EM's training can become stale, and there is little data available to assess the efficacy of training or EM preparedness.

The EMs' legacy decision-support tool, HURREVAC, is a Windows application with limited decision-support analytics and training capabilities, and it is not Web-accessible. Its

simulator has a tedious, manual entry process that requires technical knowledge of tropical cyclones, so it is rarely used.

The NHP TM effort recommended replacing HURREVAC with an integrated, Web-enabled decision-support platform that provides planning, situational awareness, data analytics, and training tools to enable EMs to make accurate, timely evacuation decisions. MIT Lincoln Laboratory began development of the new platform, *HURREVAC eXtended* (HVX), in 2015.

B. Simulator Motivation, Requirements, and Usage

One of HVX's goals is to improve EM preparedness by enabling frequent, inexpensive training on the EM's own computer, offering rapid-play serious games with never-before-seen storms, and providing training analytics for preparedness exercises. These desired properties require a storm track simulator, which plays a small but important role in HVX and is the subject of this paper.

The simulator does not need to provide the physical accuracy or fine detail of NHC supercomputer simulations and forecasting models. Rather, it needs to produce plausible storm behavior; create tracks that EMs have not seen before; accept a few simple user inputs without requiring technical tropical cyclone expertise; and run in the HVX Web application in under a minute.

To use the simulator, a user clicks on a regional map to enter a few latitude-longitude *waypoints* that outline the desired storm track. For each waypoint, the user enters the desired strength from Table I and the number of time steps until the next waypoint. Figure 1a shows an example of the user inputs. Figure 1b shows the output of the simulator, which is covered in Section III.

II. DATA, RELATED WORK, AND APPROACH

This section describes the primary data source, some related work, and the simulation approach.

A. HURDAT2 Storm Track Data

The simulator uses the HURDAT2 database [5], which contains the best track data from the NHC's post-storm analysis of actual tropical cyclones. Tables are provided for the North Atlantic Basin and the Northeast and North Central Pacific Basins. The former (5.9 MB) contains data on about 1,800 storms from 1851–2015, and the latter (3.2 MB) contains data on about 1,000 storms from 1949–2015.

For each storm, the table provides observations at synoptic time steps (0000, 0600, 1200, and 1800 UTC) and may include times of events like landfall. Each observation provides the latitude-longitude *location* of the storm center, the *strength*, and the maximum sustained *winds* in 5-knot increments. Since the 1980s, data for minimum central *pressure* in integer millibars became available, and since 2004, twelve *wind radii* describing the extent of hurricane-force (64+ knot), strong tropical storm-force (50–64 knot), and tropical storm-force (34–50 knot) winds at 5-nautical mile increments in four quadrants became available.

B. Related Work

Some aspects of the simulator are similar to work by Rumpf *et al.* [6] [7] and Nakamura *et al.* [8]. Rumpf *et al.* used a first-order Markov model and nonlinear regression and were concerned with estimating risk and losses for the insurance industry. Nakamura *et al.* reported longer-term trends in tropical cyclone behavior. To capture such trends, they concatenated randomly sampled track segments from HURDAT2, with the sampling probability based on the similarity between historical storms and the partially-formed simulated storm.

The present simulator need only generate plausible storms but must have a modest computational footprint and execute quickly. The presence of longer-term trends led us to choose a hierarchical approach, but we also wanted the ability to generate never-before-seen behavior.

C. Notation and Approach

The simulator generates results similar to HURDAT2. Let L be the total storm duration. For each synoptic time step $t = 1, 2, \ldots, L$, the simulator produces a latitude-longitude location $\boldsymbol{\ell}(t)$; forward movement $\boldsymbol{\nu}(t) = (\rho(t), \theta(t))$, where $\rho(t)$ is forward speed in knots, and $\theta(t)$ is forward heading in degrees; atmospherics (strength $s(t)$, winds $w(t)$, pressure $b(t)$); and a 12-element wind radii vector $\boldsymbol{r}(t)$.

Let M denote the number of waypoints; $\boldsymbol{w}_m$ the mth waypoint for $m = 1, \ldots, M$; s_m and t_m the strength and time step at $\boldsymbol{w}_m$, respectively; and L_m the duration or number of time steps until the next waypoint. L_m does not include the current waypoint $\boldsymbol{w}_m$, and the terminal waypoint $\boldsymbol{w}_M$ has $L_M = 0$. Hence, $L = \sum_{m=1}^{M} L_m + 1$. A *stage* $\mathcal{S}_m = \{\boldsymbol{w}_m, \boldsymbol{w}_{m+1}, s_m, s_{m+1}, L_m\}$ consists of a pair of successive waypoints, their strengths, and the stage's duration. Hence, a storm has M waypoints and $M - 1$ stages.

The indicator $I_{\text{birth}}(t)$ equals one if $t = 1$ and zero otherwise. For $\boldsymbol{w}_m$ or $\boldsymbol{\ell}(t)$, the indicator $I_{\text{land}}(\boldsymbol{w}_m)$ or $I_{\text{land}}(\boldsymbol{\ell}(t))$ equals unity if the location lies over land and zero otherwise.

We adopted a hierarchical, data-driven approach. From top to bottom, the levels are Season, Storm, Stage, and Time-step. The higher levels are mainly conceptual, as development focused on the Time-step level. The Season level models broad, seasonal characteristics such as a normal, La Niña, or El Niño weather pattern, and the number of tropical cyclones of each strength. For an individual storm, the Storm level models the number of waypoints M, birth and death locations $\boldsymbol{w}_1$ and $\boldsymbol{w}_M$, and total number of steps L. The Stage level models the remaining waypoints $\boldsymbol{w}_2, \ldots, \boldsymbol{w}_{M-1}$, and the strengths s_m and durations L_m, $m = 1, \ldots, M - 1$. For each stage $\mathcal{S}_m$, the Time-step level simulates the forward movement, atmospherics, and wind radii to produce $\boldsymbol{\ell}(t)$, $s(t)$, $w(t)$, $b(t)$, and $\boldsymbol{r}(t)$ for each time step in the stage.

The simulator was developed in MATLAB and leverages HURDAT2 to construct data-driven statistical models that can generate plausible yet never-before-seen storm behaviors. For a particular storm component, it selects a set $\mathcal{X} = \{x_1, \ldots, x_N\}$ of representative HURDAT2 data, assumes the samples
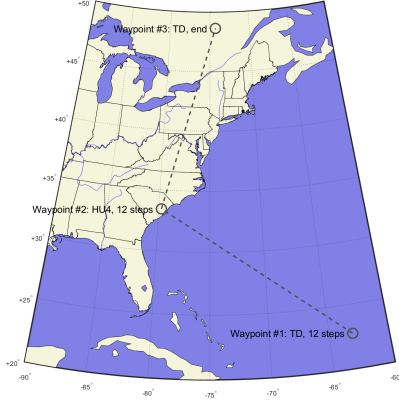
(a) Example of three user-entered waypoints with desired strengths and durations
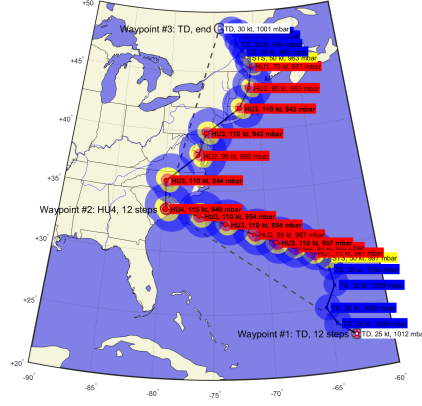
(b) Example of simulated storm track

Fig. 1. Example of simulator input and output.

x_i are distributed $f(x; \mathcal{X})$, builds a fitted model $\hat{f}(x; \mathcal{X})$, and then draws a simulated realization $\tilde{x}$ from $\hat{f}(x; \mathcal{X})$.

The approach can model dependencies that extend beyond a single time step, and it can exploit non-causal relationships. The models can model relationships explicitly, such as using a conditional distribution like $f(b(t)|w(t))$, or implicitly through the data-selection criteria used to form $\mathcal{X}$, indicated by $\mathcal{X}(w(t))$. For example, a Stage can be specified to start as a TS, end as a HU2, and endure for 10 steps (60 hours), and this information is exploited by selecting data on storms that strengthened in a similar fashion over a similar time period.

1) Kernel Density Estimation: The simulator uses both *kernel density estimation* (KDE), a non-parametric method, and parametric regression methods. Given $\mathcal{X}$, KDE approximates $f(x; \mathcal{X})$ as $\hat{f}(x; \mathcal{X}) = (1/N) \sum_i K(x, x_i)$ where $K(x, x_i)$ is a *kernel function*, typically a unimodal probability density function (PDF) with its mode at x_i. As a result, $\hat{f}(x; \mathcal{X})$ will be a valid PDF, too. A simulated realization $\tilde{x}$ is generated by drawing a sample x_i uniformly from $\mathcal{X}$, and then drawing $\tilde{x}$ from a random number generator distributed $K(x, x_i)$.

2) Regression: Regression methods assume that $f(x; \mathcal{X})$ has a parametric form $f(x; \boldsymbol{\alpha}, \mathcal{X})$, such as a normal distribution with $\boldsymbol{\alpha}$ denoting the mean and variance. They typically compute the maximum likelihood estimate $\hat{\boldsymbol{\alpha}}$ to produce the fitted model $\hat{f}(x; \mathcal{X}) = f(x; \hat{\boldsymbol{\alpha}}, \mathcal{X})$. Simulation is accomplished by drawing $\tilde{x}$ according to $f(x; \hat{\boldsymbol{\alpha}}, \mathcal{X})$.

III. SIMULATOR COMPONENT DETAILS

Figure 2 shows a block diagram of the simulator components, described in this section. For each stage, storm movement and atmospherics are simulated separately. Next, the stage results are connected into a single track. Finally, the wind radii are simulated to yield the simulated storm track.
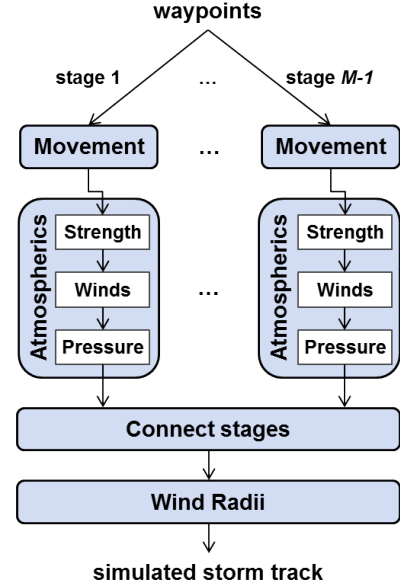

Fig. 2. Block diagram of simulator.

A. Movement

The movement simulator combines KDE with a first-order Markov model, so $f(\boldsymbol{\nu}(t)|\boldsymbol{\ell}(t), \boldsymbol{\nu}(t-1), \ldots, ; \mathcal{X}) = f(\boldsymbol{\nu}(t)|\boldsymbol{\ell}(t), \boldsymbol{\nu}(t-1); \mathcal{X})$, which is modeled implicitly as $f(\boldsymbol{\nu}(t); \mathcal{X}(\boldsymbol{\ell}(t), I_{\text{land}}(\boldsymbol{\ell}(t)), \boldsymbol{\nu}(t-1), I_{\text{birth}}(t)))$ with KDE. $\mathcal{X}$ is formed from data for locations similar to $\boldsymbol{\ell}(t)$ including over-land or -water status, forward movement similar to $\boldsymbol{\nu}(t-1)$, and similar storm birth or continuation states. The set $\mathcal{X} = \{\boldsymbol{\nu}_1, \ldots, \boldsymbol{\nu}_N\}$, where each $\boldsymbol{\nu}_i = (\rho_i, \theta_i)$ is a historical forward-movement pair. Then $\hat{f}(\boldsymbol{\nu}(t); \mathcal{X}(\boldsymbol{\ell}(t), \boldsymbol{\nu}(t-1), I_{\text{land}}(\boldsymbol{\ell}(t)), I_{\text{birth}}(t))) = (1/N) \sum_i K(\boldsymbol{\nu}(t), \boldsymbol{\nu}_i)$.
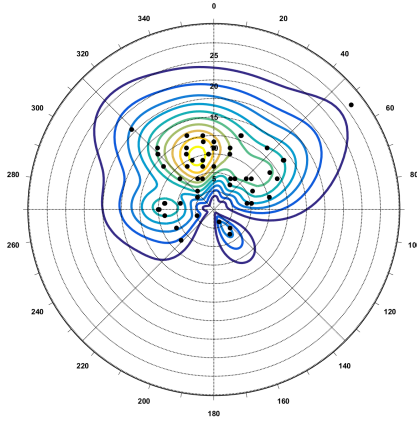
Fig. 3. Contour plot of example movement distribution with gamma-von Mises KDE from 54 historical data samples. Forward speed corresponds to radius, and forward heading corresponds to angle.

Fig. 4. Example of simulated movement for first stage.

Fig. 5. Example of KDE for maximum sustained winds given a strength of HU1 and non-birth time step.

Since forward speed is nonnegative and forward heading is directional, we adopted a product kernel $K(\boldsymbol{\nu}(t), \boldsymbol{\nu}_i) = K_g(\rho(t), \rho_i) K_{vm}(\theta(t), \theta_i)$, where K_g is a gamma kernel with 2-knot bandwidth [9], and K_{vm} is a von Mises kernel with 15-degree bandwidth. K_g is based on the gamma distribution, whose PDF has positive support and is unimodal. K_{vm} is derived from the von Mises distribution, whose PDF is circularly periodic, unimodal over a period, and described by its mode μ and concentration $\kappa > 0$. As $\kappa \to 0$, the PDF approaches a uniform distribution over all directions, and for $\kappa > 2$, the PDF approximates a Normal(μ, $\kappa^{-1/2}$) wrapped around the unit circle [10] [11] [12]. A contour plot of an example fitted distribution appears in Figure 3.

This formulation is natural, intuitive, and consistent with the idea of forward movement, and it avoids difficulties such as the nonlinear mapping between Cartesian and polar coordinates and the possibility of "negative" speed that could arise with a 2-D normal kernel often used for multi-dimensional KDE.

By itself, this model induces a random walk beginning at $\boldsymbol{\ell}(t_m) = \boldsymbol{w}_m$, so the storm is very unlikely to be located at $\boldsymbol{w}_{m+1}$ when $t = t_{m+1}$. To fix this problem, we convexly blend the KDE movement with rhumb-line movement straight to $\boldsymbol{w}_{m+1}$. As t increments from t_m to $t_{m+1} - 1$, we first compute the forward movement $\boldsymbol{\nu}'(t)$ needed to move straight from $\boldsymbol{\ell}(t)$ to $\boldsymbol{w}_{m+1}$ over the stage's remaining time steps. We then compute the blended location $\boldsymbol{\ell}(t + 1) = g^{-1}\big(\eta(t) \, g(\boldsymbol{\ell}(t) + \boldsymbol{\nu}(t)) + (1 - \eta(t)) \, g(\boldsymbol{\ell}(t) + \boldsymbol{\nu}'(t))\big)$, where g and g^{-1} denote conversion to and from Earth-centered, Earth-fixed coordinates, respectively. The weight decreases linearly from $\eta(t_m) = 1$ to $\eta(t_{m+1} - 1) = 0$, so the straight path gains influence as t approaches t_{m+1}, and the track is guaranteed to satisfy $\boldsymbol{\ell}(t_{m+1}) = \boldsymbol{w}_{m+1}$. Figure 4 shows an example of the simulated movement for a single stage.

B. Atmospherics

1) Strength: To simulate the strength $s(t)$ at each time step in stage $\mathcal{S}_m$, the simulator maps the strengths in Table I to the ordered set $\{1, 2, \ldots, 8\}$ and simply uses a noisy logistic function from s_m to s_{m+1} over duration L_m. This is the only simulator component that is not data-driven.
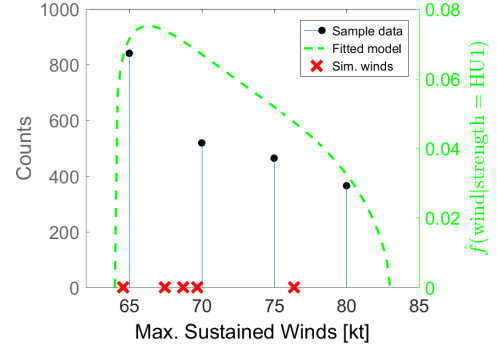
2) Winds: For a given strength $s(t)$, the winds must lie in a particular interval (see Table I). The simulator uses KDE and models $f(w(t)|s(t); \mathcal{X})$ implicitly as $\hat{f}(w(t); \mathcal{X}(s(t), I_{\text{birth}}(t)) = (1/N) \sum_i K_b(w(t), w_i)$. $\mathcal{X} = \{w_1, \ldots, w_N\}$ is formed from wind data matching the strength $s(t)$ and birth or continuation status. K_b is a beta kernel with 5-knot bandwidth [13], which remaps the beta PDF from [0,1] to an arbitrary closed interval. Storm birth must occur as a TD, TS, or STS, while continuation has eight possible strengths, yielding 11 models. An example is shown in Figure 5.

3) Pressure: To simulate pressure $b(t)$ for a stage $\mathcal{S}_m$, we assume a simple linear relation between winds and pressure [14]. Standard linear regression models $f(b(t)|w(t); \mathcal{X}(\mathcal{S}_m, I_{\text{land}}(\boldsymbol{w}_m), I_{\text{land}}(\boldsymbol{w}_{m+1})))$ as a normal distribution whose mean is a linear function of $w(t)$. The set $\mathcal{X} = \{(w_1, b_1), \ldots, (w_N, b_N)\}$ is formed from HURDAT2 storm segments that are similar to $\mathcal{S}_m$ in terms of their starting and ending strengths s_m and s_{m+1}, duration L_m, and whether the waypoints are over land or water. In this way, the simulator
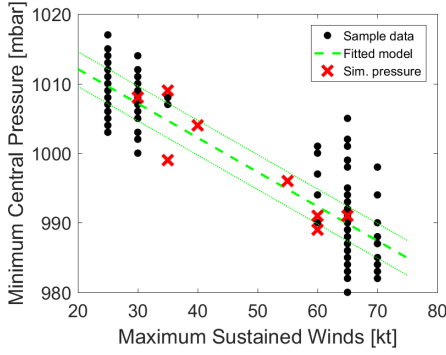
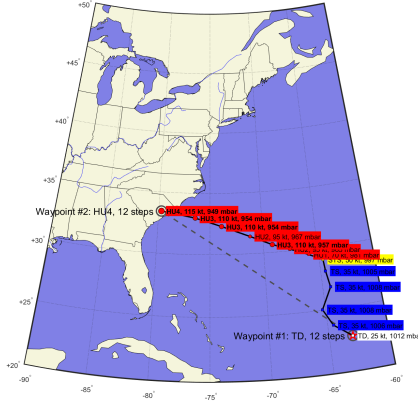Fig. 6. Example of linear regression for minimum central pressure given maximum sustained winds.

Fig. 8. Example of connected simulated stages. Wind radii have not been simulated yet.

Fig. 7. Example of simulated atmospherics for first stage.

Fig. 9. Wind radii in four quadrants.

exploits longer-term, non-causal dependencies. Data selection and linear regression are conducted online. Figure 6 displays an example fit, and Figure 7 shows the simulated strength, winds, and pressure for a single stage.

C. Stage Connection

The preceding steps simulate location $\ell(t)$ and atmospherics $s(t)$, $w(t)$, $b(t)$ for all stages and time steps. The stages are concatenated to form a track lacking only wind radii; an example appears in Figure 8.

D. Wind Radii

For each wind force ω in the ordered set $\{HU, STS, TS\}$ and each quadrant $q \in \{NE, SE, SW, NW\}$ (see Figure 9), the wind radius $r_{\omega,q}(t)$ is positive and can depend upon multiple factors, such as $w(t)$, $b(t)$, and any inner radii $r_{\omega',q}(t)$, $\omega' < \omega$. The simulator uses Poisson regression [15] to model $f(r_{\omega,q}(t)|w(t), b(t), r_{\omega',q}(t), I_{\text{land}}(\ell(t)); \mathcal{X}(\omega, q, s(t)))$. Hence, $r_{\omega,q}(t)$ is assumed to be distributed Poisson with parameter $\lambda = \boldsymbol{\alpha}^{\mathsf{T}} \boldsymbol{x}$, where $\boldsymbol{\alpha}$ is a weight vector, and $\boldsymbol{x}$ is a vector of w, b, any inner radii $r_{\omega',q}$, and over-land or -water status. $\mathcal{X}$ was selected from data matching ω, q, and strength $s(t)$, and
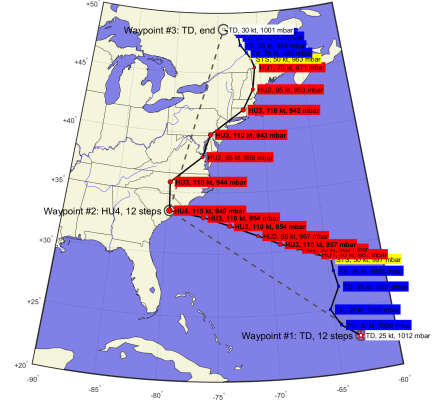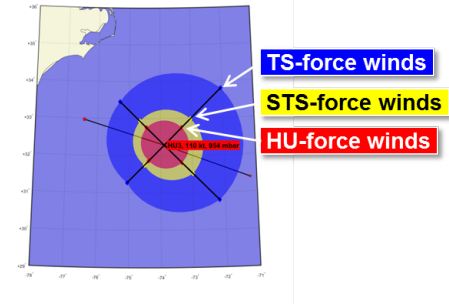
a total of 72 different models were fitted offline. Figure 10 shows example regression models for all four quadrants; the models have the circular asymmetry typical of real hurricanes.

After wind radii have been simulated for the entire track, median filtering, attenuation over land, and smoothing clean up the wind radii and complete the simulation. Figure 11 displays an example, and the final simulated storm is shown in Figure 1b.

IV. SUMMARY

The HVX storm track simulator produces plausible, data-driven storm tracks to support EM training, serious games, and training analytics. It accepts a few simple inputs and does not require detailed knowledge of tropical cyclones. The MATLAB simulator has been integrated into HVX as a Java package, and it runs in under a minute.

The mild goal of plausibility requires less stringent validation than estimating hurricane insurance risk or planning levee sites. Intermediate versions of the simulator have been positively received by the community at the 2015 Interagency Coordinating Committee on Hurricanes meeting, the 2016 National Hurricane Conference, and recent NHP WG meetings.
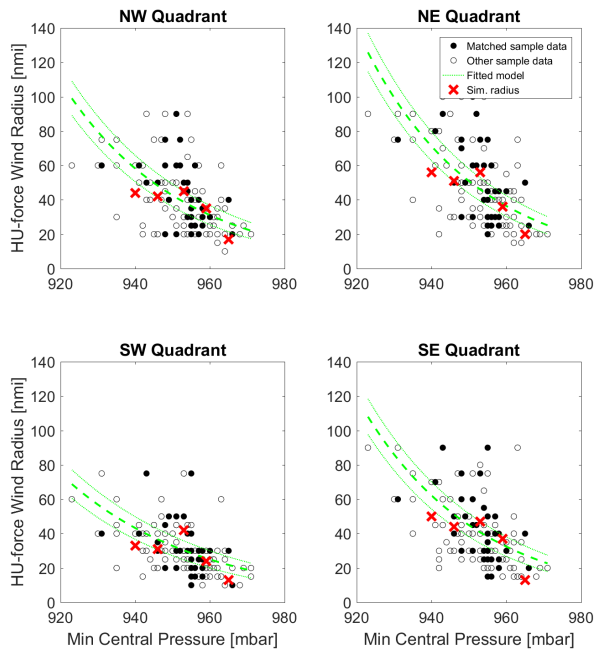
Fig. 10. Examples of Poisson regression for hurricane-force wind radii for all four quadrants given 105-knot winds, over-water status, and Category 3 hurricane strength. Hollow circles indicate sample data for other wind speeds (100- and 110-knot) or over-land status. Solid circles indicate data matching 105-knot winds and over-water status.

Fig. 11. Example of simulated wind radii. Other information (strength, winds, pressure) has been suppressed.

Current work is using simulated tracks to create forecasts, text advisories, and discussions, and to reference products such as precomputed NHC storm surge tables. Possible further technical work includes statistical validation, selection of KDE bandwidths by cross-validation, and implementation of other hierarchy levels so the user can just choose the landfall location. Finally, the modeling methods can provide the probability of different storm behaviors, so surprising events like a sudden change in forward heading or rapid strength could be injected into an otherwise ordinary simulated storm.

REFERENCES

[1] "Tropical cyclone and hurricane frequency asked questions, v. 4.9," Published online by NOAA Atlanic Oceanographic & Meteorological Laboratory, Jun. 2016, URL: http://www.aoml.noaa.gov/hrd/tcfaq/tcfaqHED.html.
[2] E. S. Blake and E. J. Gibney, "The deadliest, costliest, and most intense United States tropical cyclones from 1851 to 2010 (and other frequently requested hurricane facts)," National Hurricane Center, NOAA Technical Memorandum NWS NHC-6, Aug. 2011.
[3] T. Schott, C. Landsea, G. Hafele, J. Lorens, A. Taylor, H. Thurm, B. Ward, M. Willis, and W. Zaleski, "The Saffir-Simpson hurricane wind scale," Published online by National Hurricane Center, Feb. 2012, URL: http://www.nhc.noaa.gov/pdf/sshws.pdf.
[4] "National Hurricane Program Technology Gap Analysis Report," MIT Lincoln Laboratory, Sandia National Laboratories, Report, May 2014, sponsored by DHS S&T in coordination with FEMA.
[5] "Best track data (HURDAT2)," Published online by National Hurricane Center, Jul. 2016, URL: http://www.nhc.noaa.gov/data/?text#hurdat.
[6] J. Rumpf, H. Weindl, P. Höppe, E. Rauch, and V. Schmidt, "Stochastic modeling of tropical cyclone tracks," *Mathematical Methods Oper. Res.*, vol. 66, no. 3, pp. 475–490, Dec. 2007.
[7] ——, "Tropical cyclone hazard assessment using model-based track simulation," *Natural Hazards*, vol. 48, no. 3, pp. 383–398, Mar. 2009.
[8] J. Nakamura, U. Lall, Y. Kushnir, and B. Rajagopalan, "HITS: Hurricane intensity and track simulator with North Atlantic Ocean applications for risk assessment," *J. Applied Meteorology and Climatology*, pp. 1620–1636, Jul. 2015.
[9] S. X. Chen, "Probability density function estimation using gamma kernels," *Ann. Inst. Statist. Math.*, vol. 52, no. 3, pp. 471–480, 2000.
[10] N. I. Fisher, *Statistical Analysis of Circular Data*. Cambridge Univ. Press, 1993.
[11] G. W. Hill, "Algorithm 518: Incomplete Bessel function I_0: The von Mises distribution [S14]," *ACM Trans. Mathematical Software*, vol. 3, no. 3, pp. 279–284, 1977.
[12] P. Berens, "CircStat: A MATLAB toolbox for circular statistics," *J. Stat. Software*, vol. 31, no. 10, Sep. 2009, URL: https://www.jstatsoft.org/article/view/v031i10.
[13] S. X. Chen, "Beta kernel estimators for density functions," *Computational Statistics & Data Analysis*, vol. 31, pp. 131–145, 1999.
[14] H. E. Rosendal and S. L. Shaw, "Relationship of maximum sustained winds to minimum sea level pressure in central north pacific tropical cyclones," NOAA/NWS, Technical Memorandum NWSTM PR-24, 1982.
[15] A. J. Dobson and A. G. Barnett, *An Introduction to Generalized Linear Models*, 3rd ed. CRC Press, 2008.